SAINIK SCHOOL CHANDRAPUR
HOLIDAY HOMEWORK
Sample Paper 1
Mathematics (Code-041)
Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. The symmetric part of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 2 \\ 2 & -2 & 7 \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} 0 & -2 & -1 \\ -2 & 0 & -2 \\ -1 & -2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 2 \\ -1 & 2 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 4 & 3 \\ 4 & 8 & 0 \\ 3 & 0 & 7 \end{bmatrix}$

2. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) 1

(d) -1

3. If $f(x) = \log_e(\sin x)$, then $f'(e)$ is equal to

(a) e^{-1}

(b) e

(c) 1

(d) 0

4. The least, value of the function $f(x) = ax + b/x$, $a > 0$, $b > 0$, $x > 0$ is

(a) $\sqrt{ab}$

(b) $2\sqrt{\frac{a}{b}}$

(c) $2\sqrt{\frac{b}{a}}$

(d) $2\sqrt{ab}$

5. If $y = 2x^3 - 2x^2 + 3x - 5$, then for $x = 2$ and $\Delta x = 0.1$, value of Δy is

(a) 2.002
(b) 1.9
(c) 0
(d) 0.9

6. The value of $\int_0^1 \frac{dx}{e^x + e}$ is

(a) $\frac{1}{e}\log\left(\frac{1+e}{2}\right)$
(b) $\log\left(\frac{1+e}{2}\right)$
(c) $\frac{1}{e}\log(1+e)$
(d) $\log\left(\frac{2}{1+e}\right)$

7. The area of the region bounded by the lines $y = mx$, $x = 1$, $x = 2$ and X-axis is 6 sq units, then m is equal to

(a) 3
(b) 1
(c) 2
(d) 4

8. Find the area of a curve $xy = 4$, bounded by the lines $x = 1$ and $x = 3$ and X-axis.

(a) $\log 12$
(b) $\log 64$
(c) $\log 81$
(d) $\log 27$

9. The area of enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(a) 12 sq units
(b) 13 sq units
(c) $13\frac{1}{2}$ sq units
(d) 14 sq units

10. The degree of the differential equation
$x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...$, is

(a) 3
(b) 2
(c) 1
(d) not defined

11. The solution of $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represents a circle, when

(a) $a = b$
(b) $a = -b$
(c) $a = -2b$
(d) $a = 2b$

12. The general solution of the differential equation $\frac{dy}{dx} = e^y(e^x + e^{-x} + 2x)$ is

(a) $e^{-y} = e^x - e^{-x} + x^2 + C$
(b) $e^{-y} = e^{-x} - e^x - x^2 + C$
(c) $e^{-y} = -e^{-x} - e^x - x^2 + C$
(d) $e^y = e^{-x} + e^x + x^2 + C$

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13. If $\lambda(3\hat{i}+2\hat{j}-6\hat{k})$ is a unit vector, then the value of λ is

(a) $\pm\frac{1}{7}$ (b) ± 7

(c) $\pm\sqrt{43}$ (d) $\pm\frac{1}{\sqrt{43}}$

14. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}$ is a vector such that $\vec{a}\cdot\vec{b}=|\vec{b}|^2$ and $|\vec{a}-\vec{b}|=\sqrt{7}$, then $|\vec{b}|$ is equal to

(a) $\sqrt{7}$ (b) $\sqrt{3}$

(c) 7 (d) 3

15. The direction cosines of the line joining the points $(4,3,-5)$ and $(-2,1,-8)$ are

(a) $\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$ (b) $\left(\frac{2}{7},\frac{3}{7},\frac{-6}{7}\right)$

(c) $\left(\frac{6}{7},\frac{3}{7},\frac{2}{7}\right)$ (d) None of these

16. If the lines $\frac{1-x}{3}=\frac{y-2}{2\alpha}=\frac{z-3}{2}$ and $\frac{x-1}{3\alpha}=y-1=\frac{6-z}{5}$ are perpendicular, then the value of α is

(a) $\frac{-10}{7}$ (b) $\frac{10}{7}$

(c) $\frac{-10}{11}$ (d) $\frac{10}{11}$

17. A bag A contains 4 green and 3 red balls and bog B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B is

(a) $\frac{2}{7}$

(b) $\frac{2}{3}$

(c) $\frac{3}{7}$

(d) $\frac{1}{3}$

18. A random variable X has the probability distribution given below

X	1	2	3	4	5
$P(X=x)$	K	$2K$	$3K$	$2K$	K

Its variance is

(a) $\frac{16}{3}$

(b) $\frac{4}{3}$

(c) $\frac{5}{3}$

(d) $\frac{10}{3}$

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19. **Assertion :** $\int \frac{xe^x}{(x+1)^2}\,dx = \frac{e^x}{x+1} + C$

Reason : $\int e^x \{f(x) + f'(x)\}\,dx = e^x f(x) + C$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** $\int \frac{dx}{e^x + e^{-x} + 2} = \frac{1}{e^x + 1} + C$

Reason : $\int \frac{d\{f(x)\}}{\{f(x)\}^2} = -\frac{1}{f(x)} + C$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

22. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

23. Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where 'a' and 'b' are arbitrary constants.

24. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

25. Suppose a girls throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once gets notes whether a ‘head’ or ‘tail’ is obtained. If she obtained exactly one ‘tail’, what is the probability that she threw 3, 4 ,5 or 6 with the die?

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Let R be a relation defined on the set of natural numbers N as follow:

$$R = \{(x, y): x \in N, y \in N \text{ and } 2x + y = 24\}$$

Find the domain and range of the relation R. Also, find if R is an equivalence relation or not.

27. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.

OR

Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(adj\ A) = |A| I_3$.

28. Find the values of a and b such that the function defined as follows is continuous.

$$f(x) = \begin{cases} x+2, & x \le 2 \\ ax+b, & 2 < x < 5 \\ 3x-2, & x \ge 5 \end{cases}$$

29. Find $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$.

OR

Integrate w.r.t. x, $\frac{x^2-3x+1}{\sqrt{1-x^2}}$.

30. Solve the following differential equation

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

31. Find a vector of magnitude 5 units and parallel to the resultant of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Find the value of the following $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

OR

Find the value of the following $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

33. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to y-coordinate of the point.

OR

Find the equation of tangent to the curve $y = \frac{x-7}{x^2-5x+6}$ at the point, where it cuts the X-axis.

34. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

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OR

Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$$

and $$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k}).$$

35. A decorative item dealer deals in two items A and B. He has Rs. 15000 to invest and a space to store at the most 80 pieces. Item A costs him Rs. 300 and item B costs him Rs. 150. He can sell items A and B at respective, profit of Rs. 50 and Rs. 28. Assuming he can sell all he buys, formulate the linear programming problem in order to maximise his profit and solve it graphically.

OR

Minimise $Z = x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, x, $y \geq 0$. Show that the minimum of Z occurs at more than two points.

Section - E

Case study based questions are compulsory.

36. Sun Pharmaceutical Industries Limited is an Indian multinational pharmaceutical company headquartered in Mumbai, Maharashtra, that manufactures and sells pharmaceutical formulations and active pharmaceutical ingredients in more than 100 countries across the globe.
Sun Pharmaceutical produces three final chemical products P_1, P_2 and P_3 requiring mixup of three raw material chemicals M_1, M_2 and M_3. The per unit requirement of each product for each material (in litres) is as follows:

$$\begin{array}{cc} & \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ A = \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \\ 2 & 4 & 2 \end{bmatrix} \end{array}$$

(i) Find the total requirement of each material if the firm produces 100 litres of each product,
(ii) Find the per unit cost of production of each product if the per unit of materials M_1, M_2 and M_3 are ₹ 5, ₹ 10 and ₹ 5 respectively, and
(iii) Find the total cost of production if the firm produces 200 litres of each product.

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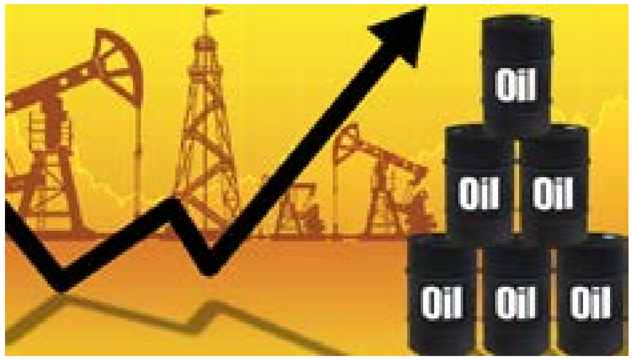
37. Commodity prices are primarily determined by the forces of supply and demand in the market. For example, if the supply of oil increases, the price of one barrel decreases. Conversely, if demand for oil increases (which often happens during the summer), the price rises. Gasoline and natural gas fall into the energy commodities category.



The price p (dollars) of each unit of a particular commodity is estimated to be changing at the rate

$$\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}}$$

where x (hundred) units is the consumer demand (the number of units purchased at that price). Suppose 400 units $(x=4)$ are demanded when the price is \$30 per unit.

(i) Find the demand function $p(x)$.

(ii) At what price will 300 units be demanded? At what price will no units be demanded?

(iii) How many units are demanded at a price of \$20 per unit?

38. Quality assurance (QA) testing is the process of ensuring that manufactured product is of the highest possible quality for customers. QA is simply the techniques used to prevent issues with product and to ensure great user experience for customers.

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A manufactured component has its quality graded on its performance, appearance, and cost. Each of these three characteristics is graded as either pass or fail. There is a probability of 0.40 that a component passes on both appearance and cost. There is a probability of 0.35 that a component passes on both performance and appearance. There is a probability of 0.31 that a component passes on all three characteristics. There is a probability of 0.64 that a component passes on performance. There is a probability of 0.19 that a component fails on all three characteristics. There is a probability of 0.06 that a component passes on appearance but fails on both performance and cost.

(i) What is the probability that a component passes on cost but fails on both performance and appearance?

(ii) If a component passes on both appearance and cost, what is the probability that it passes on all three characteristics?

(iii) If a component passes on both performance and appearance, what is the probability that it passes on all three characteristics?

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Sample Paper 2

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the identity matrix, then x is equal to

(a) -1 (b) 0

(c) 1 (d) 2

2. If $x = \frac{2\,at}{1+t^3}$ and $y = \frac{2\,at^2}{(1+t^3)^2}$, then $\frac{dy}{dx}$ is equal to

(a) ax (b) a^2x^2

(c) $\frac{x}{a}$ (d) $\frac{x}{2a}$

3. If $f(x) = \begin{cases} \frac{3\sin \pi x}{5x}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{10}$

(c) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{5}$

4. A sphere increases its volume at the rate of π cm^3/s. The rate at which its surface area increases, when the radius is 1 cm is

(a) 2π sq cm/s (b) π sq cm/s

(c) $\frac{3\pi}{2}$ sq cm/s (d) $\frac{\pi}{2}$ sq cm/s

5. The slope of the normal to the curve $y = x^2 - \frac{1}{x^2}$ at $(-1, 0)$ is

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) 4 (d) -4

6. If $\int_0^a f(2a - x)\,dx = m$ and $\int_0^a f(x)\,dx = n$, then $\int_0^{2a} f(x)\,dx$ is equal to

(a) $2m + n$ (b) $m + 2n$

(c) $m - n$ (d) $m + n$

7. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\,dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

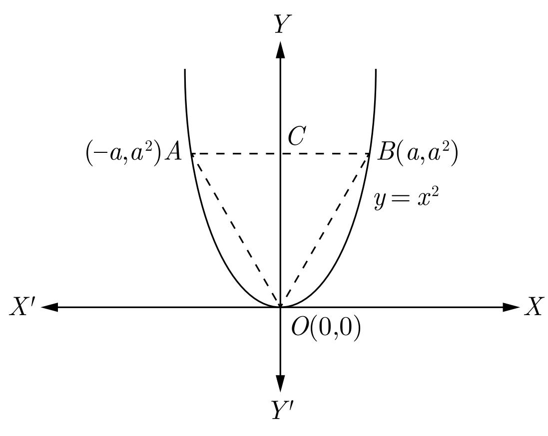
(c) $\frac{\pi}{2}$ (d) π

8. $3a\int_0^1 \left(\frac{ax - 1}{a - 1}\right)^2 dx$ is equal to

(a) $a - 1 + (a - 1)^{-2}$ (b) $a + a^{-2}$

(c) $a - a^2$ (d) $a^2 + \frac{1}{a^2}$

9. The given figure shows a $\Delta\,AOB$ and the parabola $y = x^2$. The ratio of the area of the $\Delta\,AOB$ to the area of the region AOB of the parabola $y = x^2$ is equal to



(a) $\frac{3}{5}$ (b) $\frac{3}{4}$

(c) $\frac{7}{8}$ (d) $\frac{5}{6}$

10. The area bounded by $y = |\sin x|$, X-axis and the lines $|x| = \pi$ is

(a) 2 sq units (b) 3 sq units

(c) 4 sq units (d) None of these

11. Order of the equation $\left(1+5\frac{dy}{dx}\right)^{3/2}=10\frac{d^3y}{dx^3}$ is

(a) 2 (b) 3

(c) 1 (d) 0

12. $y=2e^{2x}-e^{-x}$ is a solution of the differential equation

(a) $y_2+y_1+2y=0$ (b) $y_2-y_1+2y=0$

(c) $y_2+y_1=0$ (d) $y_2-y_1-2y=0$

13. The projection of $\vec{a}=3\hat{i}-\hat{j}+5\hat{k}$ on $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$ is

(a) $\frac{8}{\sqrt{35}}$ (b) $\frac{8}{\sqrt{39}}$

(c) $\frac{8}{\sqrt{14}}$ (d) $\sqrt{14}$

14. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. Then, which one of the following is correct?

(a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=0$

(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq 0$

(c) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=0$

(d) $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$ are mutually perpendicular.

15. The angle between the lines $x=1,\ y=2$ and $y=-1,\ z=0$ is

(a) 30° (b) 60°

(c) 90° (d) 0°

16. The line joining the points $(1,1,2)$ and $(3,-2,1)$ meets the planes $3x+2y+z=6$ at the point

(a) $(1,1,2)$ (b) $(3,-2,1)$

(c) $(2,-3,1)$ (d) $(3,2,1)$

17. For two events A and B, if $P(A)=P\left(\frac{A}{B}\right)=\frac{1}{4}$ and $P\left(\frac{B}{A}\right)=\frac{1}{2}$, then

(a) A and B are independent events

(b) $P\left(\frac{A'}{B}\right)=\frac{3}{4}$

(c) $P\left(\frac{B'}{A}\right)=\frac{1}{2}$

(d) All of the above

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18. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$, then $P\left(\frac{A'}{B}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

19. **Assertion :** The matrix A $= \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is an orthogonel matrix.

Reason : If A and B are orthagonal, then AB is also orthegonal.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** If A $= \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj (adj A) $=$ A.

Reason : $|\text{adj}\,(\text{adj}\,\text{A})| = |\text{A}|^{(n-1)^2}$, where A be n rowed non-singular matrix.

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Show that the function $f(x) = x^3 - 3x^2 + 3x$, $x \in R$ is increasing on R.

22. Find the differential equation representing the family of curves $y = ae^{bx+5}$, where a and b are arbitrary constants.

23. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

OR

The x-coordinate of point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate.

24. Maximize $Z = x + y$,
subject to $x - y \leq -1$, $x + y \leq 0$, x, $y \geq 0$.

25. If $P(not\ A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Consider $f : R \to R$ given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

OR

Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

27. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

28. Find the equations of tangent and normal to the curve $x = 1 - \cos\theta$, $y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$.

29. Find $\int \frac{(2x-5)\, e^{2x}}{(2x-3)^3}\, dx$.

OR

Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$.

30. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

31. Maximise and minimise $Z = x + 2y$ subject to the constraints

$$\begin{aligned} x + 2y &\geq 100 \\ 2x - y &\leq 0 \\ 2x + y &\leq 200 \\ x,\ y &\geq 0 \end{aligned}$$

Solve the above LPP graphically.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b) R (c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.

33. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$, $x^2 \leq 1$, then find dy/dx.

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OR

Find whether the following function is differentiable at $x = 1$ and $x = 2$ or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \leq x \leq 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

34. Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$, using method of integration.

OR

Using integration, find the area of the circle $x^2 + y^2 = 16$, which is exterior to the parabola $y^2 = 6x$.

35. Using vectors, find the area of the ΔABC, whose vertices are $A(1,2,3)$, $B(2,-1,4)$ and $C(4,5,-1)$.

OR

If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k and hence, find the equation of the plane containing these lines.

Section - E

Case study based questions are compulsory.

36. Pastry is a dough of flour, water and shortening that may be savoury or sweetened. Sweetened pastries are often described as bakers' confectionery. The word "pastries" suggests many kinds of baked products made from ingredients such as flour, sugar, milk, butter, shortening, baking powder, and eggs.

The Sunrise Bakery Pvt Ltd produces three basic pastry mixes A, B and C. In the past the mix of ingredients has shown in the following matrix:

$$\begin{array}{cc} & \begin{array}{ccc} \text{Flour} & \text{Fat} & \text{Sugar} \end{array} \\ \text{Type} \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 5 & 1 & 1 \\ 6.5 & 2.5 & 0.5 \\ 4.5 & 3 & 2 \end{bmatrix} \end{array}$$

(All quantities in kg)

Due to changes in the consumer's tastes it has been decided to change the mixes using the following amendment matrix:

$$\begin{array}{cc} & \begin{array}{ccc} \text{Flour} & \text{Fat} & \text{Sugar} \end{array} \\ \text{Type} \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \end{bmatrix} \end{array}$$

Continue on next page.....

OR

Using matrix algebra you are required to calculate:

(i) the matrix for the new mix:

(ii) the production requirement to meet an order for 50 units of type A, 30 units of type B and 20 units of type C of the new mix;

(iii) the amount of each type that must be made to totally use up 370 kg of flour, 170 kg of fat and 80 kg of sugar that are at present in the stores.

37. Brine is a high-concentration solution of salt in water. In diverse contexts, brine may refer to the salt solutions ranging from about 3.5% up to about 26%. Brine forms naturally due to evaporation of ground saline water but it is also generated in the mining of sodium chloride.

A tank initially contains 10 gallons of pure water. Brine containing 3 pounds of salt per gallon flows into the tank at a rate of 2 gallons per minute, and the well-stirred mixture flows out of the tank at the same rate.

(i) How much salt is present at the end of 10 minutes?

(ii) How much salt is present in the long run?

Continue on next page.....

38. ICAR-Indian Agricultural Research Institute is an autonomous body responsible for co-ordinating agricultural education and research in India. It reports to the Department of Agricultural Research and Education, Ministry of Agriculture. The Union Minister of Agriculture serves as its president. It is the largest network of agricultural research and education institutes in the world.

ICAR grows vegetables and grades each one as either good or bad for its taste, good or bad for its size, and good or bad for its appearance. Overall 78% of the vegetables have a good taste. However, only 69% of the vegetables have both a good taste and a good size. Also, 5% of the vegetables have both a good taste and a good appearance, but a bad size. Finally, 84% of the vegetables have either a good size or a good appearance.

(i) If a vegetable has a good taste, what is the probability that it also has a good size?

(ii) If a vegetable has a bad size and a bad appearance, what is the probability that it has a good taste?

□□□□□□□

Sample Paper 3

Mathematics (Code-041)

Class XII Session 2023-24

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub-parts.

Section - A

Multiple Choice Questions each question carries 1 mark.

1. If A and B are two equivalence relations defined on set C, then

(a) $A \cap B$ is an equivalence relation

(b) $A \cap B$ is not an equivalence relation

(c) $A \cap B$ is an equivalence relation

(d) $A \cap B$ is not an equivalence relation

2. If A and B are two symmetric matrices of same order. Then, the matrix $AB - BA$ is equal to

(a) a symmetric matrix

(b) a skew-symmetric matrix

(c) a null matrix

(d) the identity matrix

3. If $x = e^{y + e^{y + e^{y + \dots}}}$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{1}{x}$

(b) $\dfrac{1-x}{x}$

(c) $\dfrac{x}{1+x}$

(d) None of these

4. The derivative of $\log|x|$ is

(a) $\dfrac{1}{x}$, $x > 0$

(b) $\dfrac{1}{|x|}$, $x \neq 0$

(c) $\dfrac{1}{x}$, $x \neq 0$

(d) None of these

5. The condition that $f(x) = ax^3 + bx^2 + cx + d$ has no extreme value is

(a) $b^2 > 3ac$

(b) $b^2 = 4ac$

(c) $b^2 = 3ac$

(d) $b^2 < 3ac$

6. Which of the following function is decreasing on $(0, \pi/2)$?

(a) $\sin 2x$ (b) $\cos 3x$

(c) $\tan x$ (d) $\cos 2x$

7. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x} dx$ is equal to

(a) $-\log(1+\sin^2 x)+C$ (b) $\log(1+\cos^2 x)+C$

(c) $-\log(1+\cos^2 x)+C$ (d) $\log(1+\tan^2 x)+C$

8. The value of $\int_{-2}^{2}(x\cos x + \sin x + 1)\,dx$ is

(a) 2 (b) 0

(c) -2 (d) 4

9. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

10. The area bounded by the curve $y = \frac{1}{2}x^2$, the X-axis and the ordinate $x = 2$ is

(a) $\frac{1}{3}$ sq unit (b) $\frac{2}{3}$ sq unit

(c) 1 sq unit (d) $\frac{4}{3}$ sq unit

11. The solution of $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represents a circle, when

(a) $a = b$ (b) $a = -b$

(c) $a = -2b$ (d) $a = 2b$

12. The family of curves $y = e^{a\sin x}$, where a is an arbitrary constant is represented by the differential equation

(a) $\log y = \tan x \frac{dy}{dx}$ (b) $y\log y = \tan x \frac{dy}{dx}$

(c) $y\log y = \sin \frac{dy}{dx}$ (d) $\log y = \cos x \frac{dy}{dx}$

13. The order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x}$, is

(a) 3 (b) 1

(c) 2 (d) 4

14. If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|$ is equal to

(a) 16 (b) 8

(c) 3 (d) 12

15. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\vec{x}\cdot\vec{y}=0$, then

(a) $|\vec{x}+\vec{y}|=1$ (b) $|\vec{x}+\vec{y}|=\sqrt{3}$

(c) $|\vec{x}+\vec{y}|=2$ (d) $|\vec{x}+\vec{y}|=\sqrt{2}$

16. The distance of the plane $6x-3y+2z-14=0$ from the origin is

(a) 2 (b) 1

(c) 14 (d) 8

17. If $P(A\cup B)=0.83$, $P(A)=0.3$ and $P(B)=0.6$, then the events will be

(a) dependent (b) independent

(c) cannot say anything (d) None of the above

18. If A and B are two independent events such that $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$, then P (neither A nor B) is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$

(c) $\frac{5}{6}$ (d) $\frac{1}{3}$

19. Let us define $\tan^{-1}\mathrm{A}+\tan^{-1}\mathrm{B}=\tan^{-1}\left(\frac{\mathrm{A}+\mathrm{B}}{1-\mathrm{AB}}\right)$

Assertion : The value of $\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)$ is $\frac{\pi}{4}$.

Reason : If $x>0, y>0$ than $\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y-x}{y+x}\right)=\frac{\pi}{4}$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

20. **Assertion :** If A is a matrix of order 2×2, then $|\mathrm{adj}\,\mathrm{A}|=|\mathrm{A}|$
Reason : $|\mathrm{A}|=|\mathrm{A}^{\mathrm{T}}|$

(a) Assertion is true, reason is true, reason is a correct explanation for assertion.

(b) Assertion is true,reason is true, reason is not a correct explanation for assertion.

(c) Assertion is true, reason is false.

(d) Assertion is false, reason is true.

Section - B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Write the vector equation of a line passing through point $(1,-1,2)$ and parallel to the line whose equation is $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$.

22. Write the value of $\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$.

23. Find the differential equation representing the family of curves $y=ae^{bx+5}$, where a and b are arbitrary constants.

24. Write a unit vector in the direction of the sum of the vectors $\vec{a}=2\hat{i}+2\hat{j}-5\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-7\hat{k}$.

OR

If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ, so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors.

25. Two groups are computing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product introduced way by the second group.

Section - C

This section comprises of short answer type questions (SA) of 3 marks each.

26. If $A=\{1,2,3\}, B=\{4,5,6,7\}$ and
$f=\{(1,4),(2,5),(3,6)\}$ is a function from A to B. State whether f is one-one or not.

27. Solve for x, $\cos^{-1}x+\sin^{-1}\left(\frac{x}{2}\right)=\frac{\pi}{6}$.

28. Given $A=\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1}=9I-A$.

OR

If $A=\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1}=kA$, then find the value of k.

29. Evaluate $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx$.

30. Evaluate $\int_0^{\pi/2} x^2 \sin x\,dx$.

OR

Evaluate $\int_{\pi/4}^{\pi/2} \cos 2x \cdot \log(\sin x)\,dx$.

31. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

Section - D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then find the values of a and b.

OR

Find the adjoint of the matrix $A=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$ and hence show that $A(adj\ A)=|A|I_3$.

33. If $x=\cos t+\log\tan\left(\frac{t}{2}\right)$, $y=\sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$.

OR

Find the values of a and b, if the function f defined by $f(x)=\begin{cases}x^2+3x+a, & x\leq 1\\ bx+2, & x>1\end{cases}$ is differentiable at $x=1$.

34. Find the particular solution of the differential equation satisfying the given condition.

$$x^2\,dy+(xy+y^2)\,dx\ =0,\ \text{when}\ y(1)=1$$

OR

Find the particular solution of the differential equation

$$x\frac{dy}{dx}-y+x\operatorname{cosec}\left(\frac{y}{x}\right)\ =0$$

35. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}\cdot\vec{c}=18$.

Section - E

Case study based questions are compulsory.

36. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20% of the population is accident prone.

On the basis of above information, answer the following questions.

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy?
(ii) Suppose that a new policy holder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

37. Chemical reaction, a process in which one or more substances, the reactants, are converted to one or more different substances, the products. Substances are either chemical elements or compounds. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as products.

In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance present at any time t. Initially $(t = 0)$ 50 g of the first substance was present; 1 hr later, only 10 g of it remained.
(i) Find an expression that gives the amount of the first substance present at any time t.
(ii) What is the amount present after 2 hr?

38. Vitamins are nutritional substances which you need in small amounts in your diet. Vitamins A and E are fat-soluble vitamins, meaning they're stored in your body's fat cells, but they need to have their levels topped up regularly. Vitamin C is a water-soluble vitamin found in citrus and other fruits and vegetables, and also sold as a dietary supplement. It is used to prevent and treat scurvy. Vitamin C is an essential nutrient involved in the repair of tissue, the formation of collagen, and the enzymatic production of certain neurotransmitters.

A dietician wishes to mix two types of foods in such a way that the vitamin contents of mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C, while food II contains 1 unit per kg of vitamin A an 2 units per kg of vitamin C. It costs Rs. 30 per kg to purchase food I and Rs. 42 per kg to purchase food II.
(i) Formulate above as an LPP and solve it graphically.
(ii) Find the minimum cost of such a mixture.